Math 1B: Worksheet 2

March 2

For each of the following problems, write a differential equation to describe the physical system, solve it and use the solution to address the queries.

1. A large tank is partially filled with 100 gallons of fluid in which 10 pounds of salt is dissolved. A highly concentrated salt solution containing $\frac{1}{2}$ pound of salt per gallon is pumped into the tank at a rate of 6 gal/min. The well-mixed solution is then pumped out at a slower rate of 4 gal/min. Find the number of pounds of salt in the tank after 30 minutes.

2. The population of a town grows at a rate proportional to the population present at time t. The initial population of 500 increases by 15% in 10 years. What will be the population in 30 years?

3. A dead body was found within a closed room of a house where the temperature was a constant 70° F. At the time of discovery the core temperature of the body was determined to be 85° F. One hour later a second measurement showed that the core temperature of the body was 80° F. Assume that the core temperature of the body at the beginning was 98.6° F and that the cooling process obeys Newton's Law of Cooling. Determine how many hours elapsed before the body was found.

4. A sailboat weighing a ton is running along a straight course with the wind providing a constant forward force of 500 N. The only other force acting on the boat is resistance as the boat moves through the water. The resisting force (in units of N) is numerically equal to five times the boat's speed, and the initial velocity is 1 ms^{-1}. What is the maximum velocity (in ms^{-1}) of the boat under this wind?

5. A vat with 500 gallons of beer contains 4% alcohol (by volume). Beer with 6% alcohol is pumped into the vat at a rate of 5 gal/min and the mixture is pumped out at the same rate. What is the percentage of alcohol after an hour?